Simulations of Binary Black Hole Mergers for LISA

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Outline

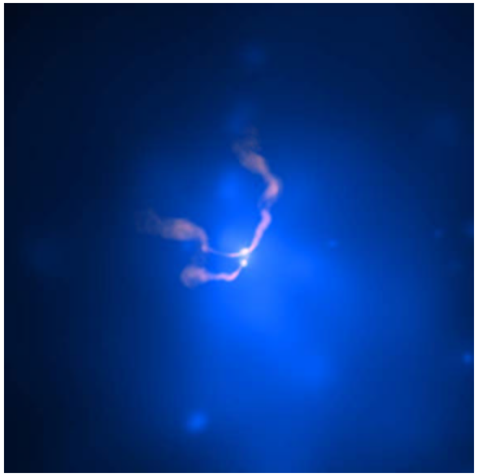
- Introduction
- Methods
- Results

Introduction

- Among the key predictions of Einstein's general theory of relativity are black holes and gravitational waves.
- Black holes are formed under physical situations in which gravitational forces are *extremely* strong.
- Gravitational waves are expected to be produced in physical scenarios such as merger of two black holes, or supernovae explosions. They are analogous to electromagnetic radiation in many ways, but much harder to detect.
- As early as 1917, Einstein recognized their existence, but, in year 2006, gravitational waves have yet to be directly detected.

Introduction

- Super-Massive black holes (black holes with $\sim 10^5 - 10^9$ solar mass) are believed to be central engines of most active galatic nuclei and play significant roles in formation and evolution of galaxies and various dynamical phenomena such as jets.
- Following galactic merger, a binary black hole system will be formed that can eventaully merge into a single black hole emitting gravitational waves.



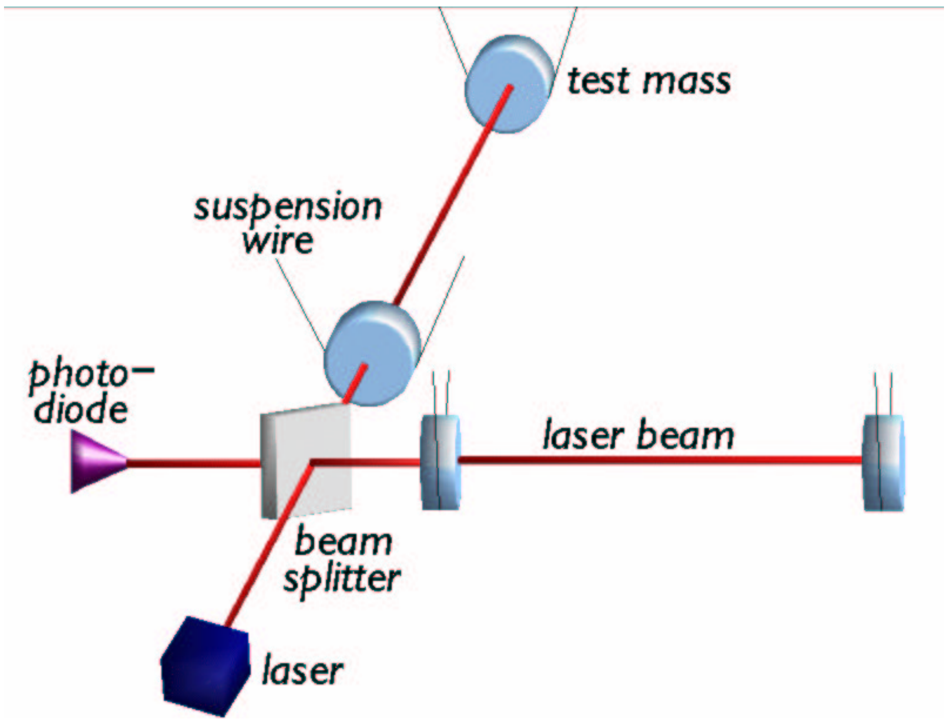
Composite X-ray(blue) and Radio (pink) image of Abell 400 galaxy cluster

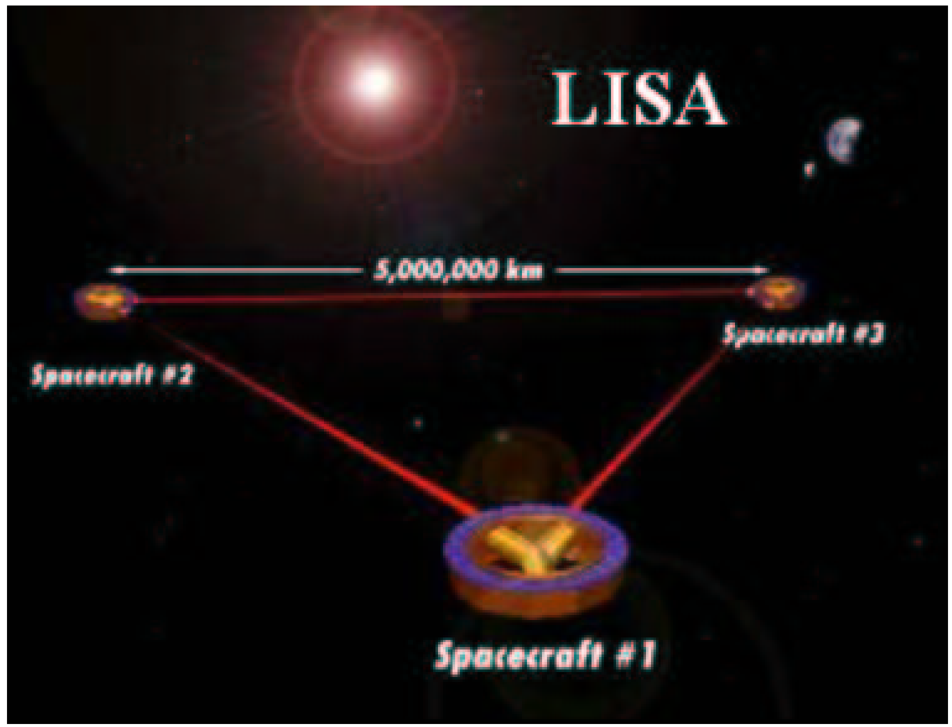
Introduction

- Gravitataional waves can probe deep into the source regions and convey direct information about source dynamics and spacetime geometry for which electromagnetic signals are in general not available.
- Black hole binary mergers are among the most anticipated sources of gravtiational wave observatories such as LISA and LIGO.
- The complex nature of the merger problem necessitate a computational approach. We use large high performance computers to simulate processes in black hole interactions and then attempt to extract the salient physics from the results of the simulations.

Introduction

- Roughly speaking, distance travelled by laser beams between the test masses change when gravitational waves pass through. Detectors will measure strain amplitude, $h = \frac{\Delta L}{L}$.





Methods

- $ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$
- A version of BSSN system of equations with variables $\{\tilde{\gamma}_{ij}, \phi, \tilde{A}_{ij}, K, \tilde{\Gamma}^i\}$ defined from the usual ADM variables $\{\gamma_{ij}, K_{ij}\}$.

$$
\begin{aligned}
\phi &= \frac{1}{12}\log\gamma \\
K &= \gamma^{ab}K_{ab} \\
\tilde{\gamma}_{ij} &= e^{-4\phi}\gamma_{ij} \\
\tilde{A}_{ij} &= e^{-4\phi}\left(K_{ij} - \frac{1}{3}\gamma_{ij}K\right) \\
\tilde{\Gamma}^i &= \tilde{\gamma}^{ab}\tilde{\Gamma}^i_{ab}
\end{aligned}
$$

where $\tilde{\Gamma}^i_{ab}$ Christoffel symbol associated with the conformal metric $\tilde{\gamma}_{ij}$.

- $\{\tilde{\gamma}_{ij} \equiv \tilde{\gamma}_{ij}(t,x,y,z), \phi \equiv \phi(t,x,y,z), \tilde{A}_{ij} \equiv \tilde{A}_{ij}(t,x,y,z), K \equiv K(t,x,y,z), \tilde{\Gamma}^i \equiv \tilde{\Gamma}^i(t,x,y,z)\}$

Methods

- Equation of motion for $\{\tilde{\gamma}_{ij}, \phi, \tilde{A}_{ij}, K, \tilde{\Gamma}^i\}$

$$
\begin{aligned}
\frac{d\phi}{dt} &= -\frac{1}{6}\alpha K \\
\frac{dK}{dt} &= -\nabla^a\nabla_a\alpha + \alpha\left(\tilde{A}_{ab}\tilde{A}^{ab} + \frac{1}{3}K^2\right) \\
\frac{d\tilde{\gamma}_{ij}}{dt} &= -2\alpha\tilde{A}_{ij} \\
\frac{d\tilde{A}_{ij}}{dt} &= e^{-4\phi}\left(-\nabla_i\nabla_j\alpha + \alpha R_{ij}\right)^{\mathrm{TF}} + \alpha\left(K\tilde{A}_{ij} - 2\tilde{A}_{ia}\tilde{A}^a{}_j\right) \\
\frac{\partial\tilde{\Gamma}^i}{\partial t} &= 2\alpha\left(\tilde{\Gamma}^i_{ab}\tilde{A}^{ab} - \frac{2}{3}\tilde{\gamma}^{ia}K_{,a} + 6\tilde{A}^{ia}\phi_{,a}\right) \\
&\quad - \tilde{\Gamma}^j\beta^i{}_{,j} + \frac{2}{3}\tilde{\Gamma}^i\beta^j{}_{,j} + \beta^k\tilde{\Gamma}^i{}_{,k} \\
&\quad + \tilde{\gamma}^{jk}\beta^i{}_{,jk} + \frac{1}{3}\tilde{\gamma}^{ij}\beta^k{}_{,kj} - 2\tilde{A}^{ia}\alpha_{,a} - (\chi_{yo} + \frac{2}{3})\left(\tilde{\Gamma}^i - \tilde{\gamma}^{kl}\tilde{\Gamma}^i_{kl}\right)\beta^m_{,m}
\end{aligned}
$$

where $d/dt = \partial/\partial t - \mathcal{L}_\beta$. The last term in $\tilde{\Gamma}^i$ equation suggested by Yo et al to suppress exponential growth of $\tilde{\Gamma}^i$ when $\beta^j{}_{,j} > 0$.

Methods

- Constraint equations are solved only at $t = 0$ to set up initial data.
- Initial data: Assume conformal flatness and maximal slicing ($\tilde{\gamma}_{ij} = \eta_{ij}, K = 0$)
- Take Bowen York form of extrinsic curvature

$$K^{ij} = \frac{3}{2r^2}(P^i n^j + P^j n^i - (\gamma^{ij} - n^i n^j)P^k n_k) + \frac{3}{r^3}(\epsilon^{ikl} S_k n_l n^j + \epsilon^{jkl} S_k n_l n^i)$$

- Puncture method: split $\phi = \phi_{BL} + u, \phi_{BL} = 1 + \sum_{n=1}^{2} \frac{m_n}{2|\vec{r} - \vec{r_n}|}$ where the nth black hole has mass (parameter) m_n and is located at coordinate $\vec{r_n}$. We solve HCE for u using MultiGrid algorithm.

$$\Delta u + \beta(1 + \frac{u}{\phi_{BL}})^{-7} = 0, \beta = \frac{1}{8}\phi_{BL}^{-7} K^{ij} K_{ij}$$

- For $t > 0$, we directly finite-difference the whole ϕ w/o making the split. May generate non-convergence or lower-order convergence near “punctures”. In practice, puncture “errors” do not influence the dynamics outside the horizon. Combined with proper choices of gauges, this strategy is proven to be a robust way to realize moving black hole idea without a need for excision technique. (Hannam et al, 2006)

Methods

- Gauge conditions do NOT change dynamics, but turn out to be crucial in getting stable numerical evolution.
- Gauge conditions: specify α, β^i. We currently use the following conditions (van Meter, Baker, Koppitz, Choi, PRD, 2006)

$$
\begin{aligned}
\partial_t \alpha &= -2\alpha K + \beta^i \partial_i \alpha \\
\partial_t \beta^i &= \frac{3}{4} B^i + \beta^j \partial_j \beta^i \\
\partial_t B^i &= \partial_t \tilde{\Gamma}^i - \beta^j \partial_j (\tilde{\Gamma}^j - B^i) - \eta B^i
\end{aligned}
$$

where η is a constant typically between $(1, 2)$.

Waveform Analysis

- Use NP Weyl tensor component Ψ_4 to analyse (outgoing) gravitational wave content.
- Harmonic decomposition

$$\Psi_4(r,\theta,\phi,t) = \sum_{lm} A_{lm}(r,t)\,{}_{-2}Y_{lm}(\theta,\phi)$$

$$A_{lm}(r,t) = \int \Psi_4(r,\theta,\phi,t)\,{}_{-2}Y_{lm}(\theta,\phi) d\Omega$$

- Given Ψ_4, one can calculate E, J_z, P_z.

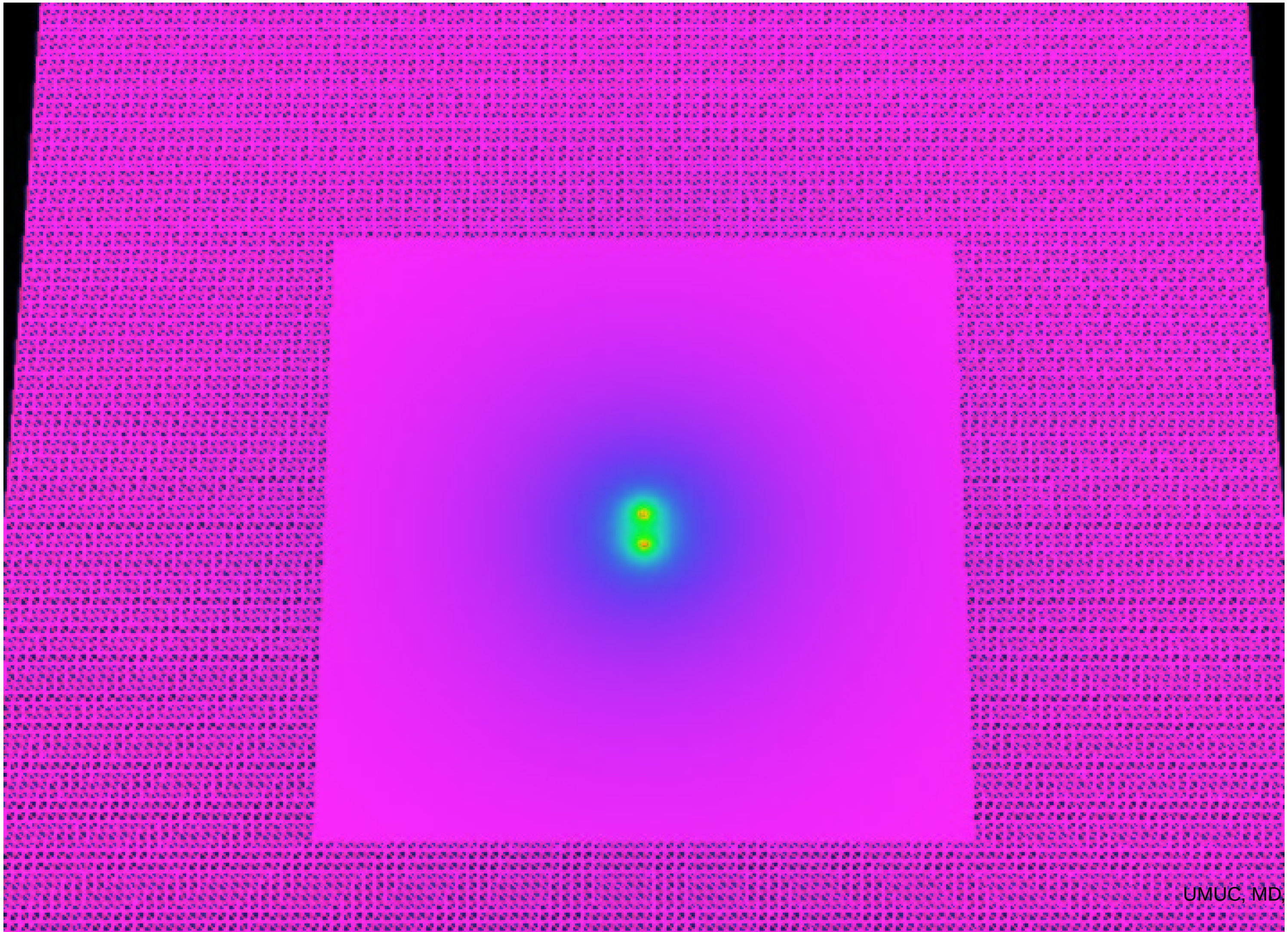
$$\begin{aligned}
E &= \frac{r^2}{4\pi}\int\int_\Omega \left|\int_{-\infty}^{t} dt' \Psi_4(t',r,\theta,\phi)\right|^2 d\Omega dt \\
P_z &= \frac{r^2}{4\pi}\int\int_\Omega \cos\theta \left|\int_{-\infty}^{t} dt' \Psi_4(t',r,\theta,\phi)\right|^2 d\Omega dt \\
J_z &= -\frac{r^2}{4\pi}\int Re[\int_\Omega d\Omega (\partial_\phi \int_{-\infty}^{t} dt' \psi_4(t',r,\theta,\phi)) \\
&\times \left(\int dt' \int d\tilde{t}\bar{\psi}_4(\tilde{t},r,\theta,\phi)\right)]dt
\end{aligned}$$

Some details on numerical methods

- Einstein equations consist of a set of coupled nonlinear partial differential equations of mixed hyperbolic-elliptic types.
- We distretize Einstein equations on a "3D"-computational grid using finite difference method with Runge-Kutta time integrator.
- Adaptive mesh refinement, which is needed to properly handle a variety of length scales in the problem, is a crucial component of our code.
- We use `PARAMESH` package to implement parallelism and adaptivity.
- Scaling performance is good $\sim 86(67)\%$ level up to 1016(2032) CPUs for the full AMR simultaions (on Project Columbia, NASA Ames).

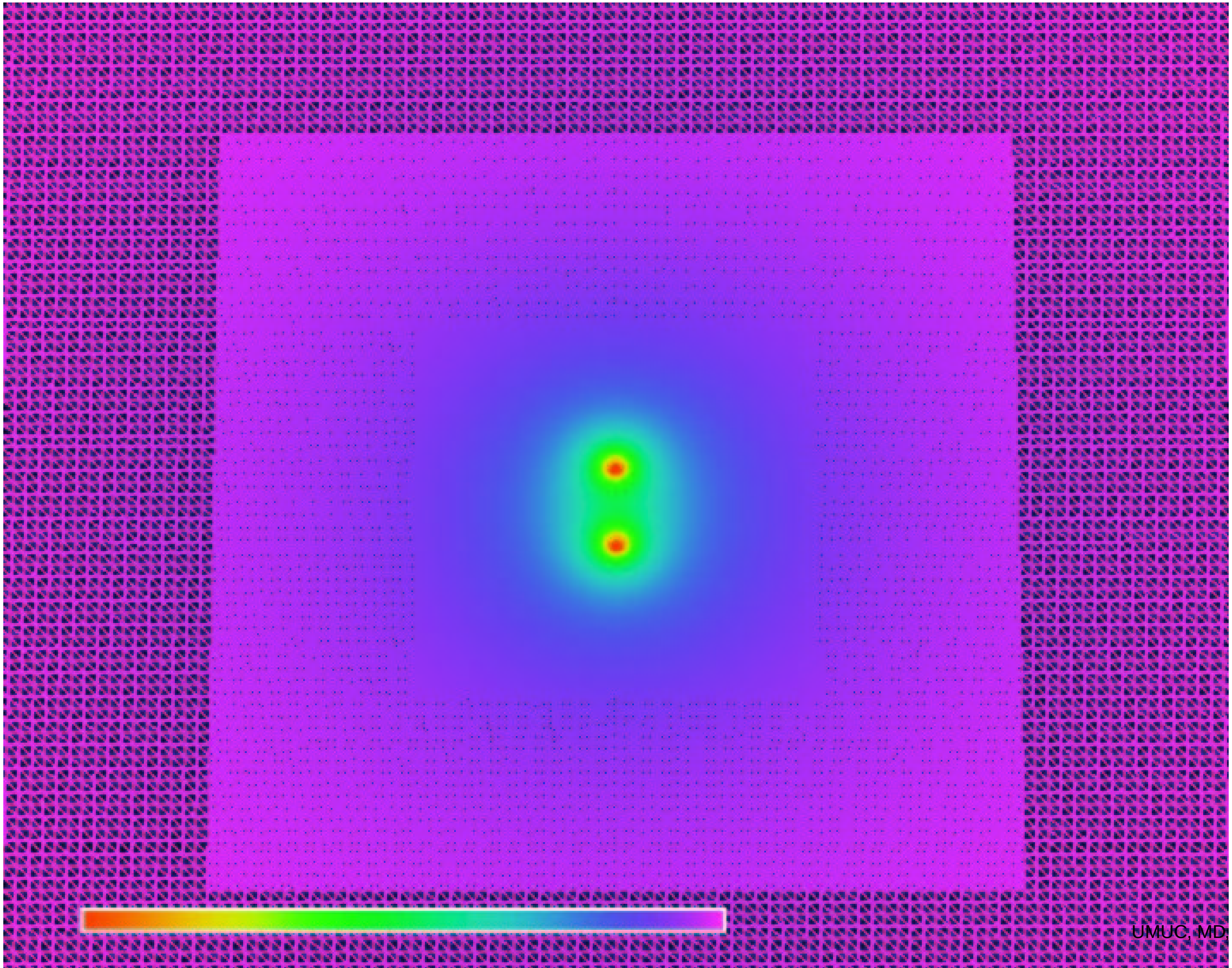
Computational grid structure

- Gravitational "potential" on $z = 0$ slice: Size of the domain is $256M$ compared to "size" of black hole $\sim M$.

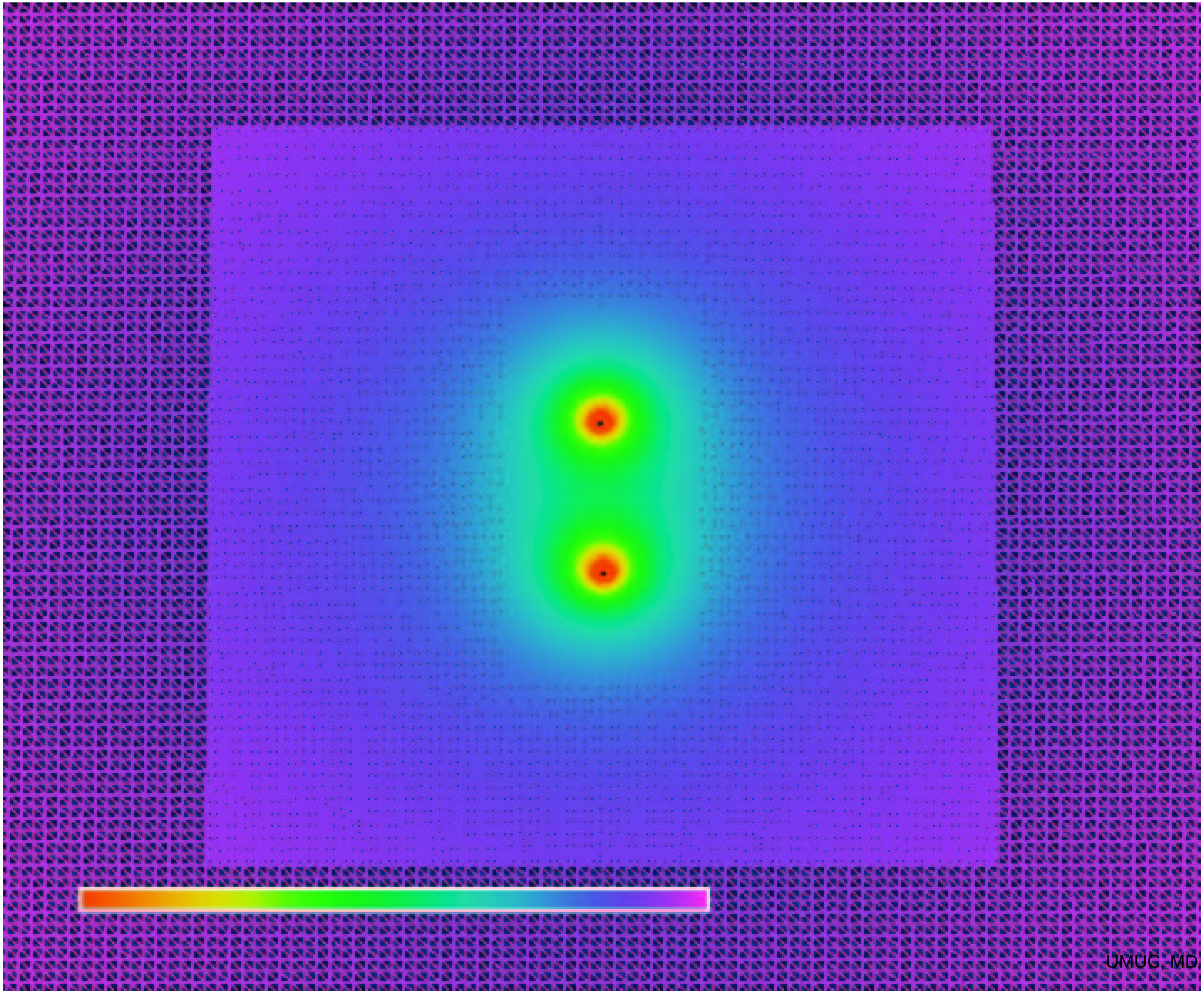


Computational grid structure

- Zoom-in view



Computational grid structure



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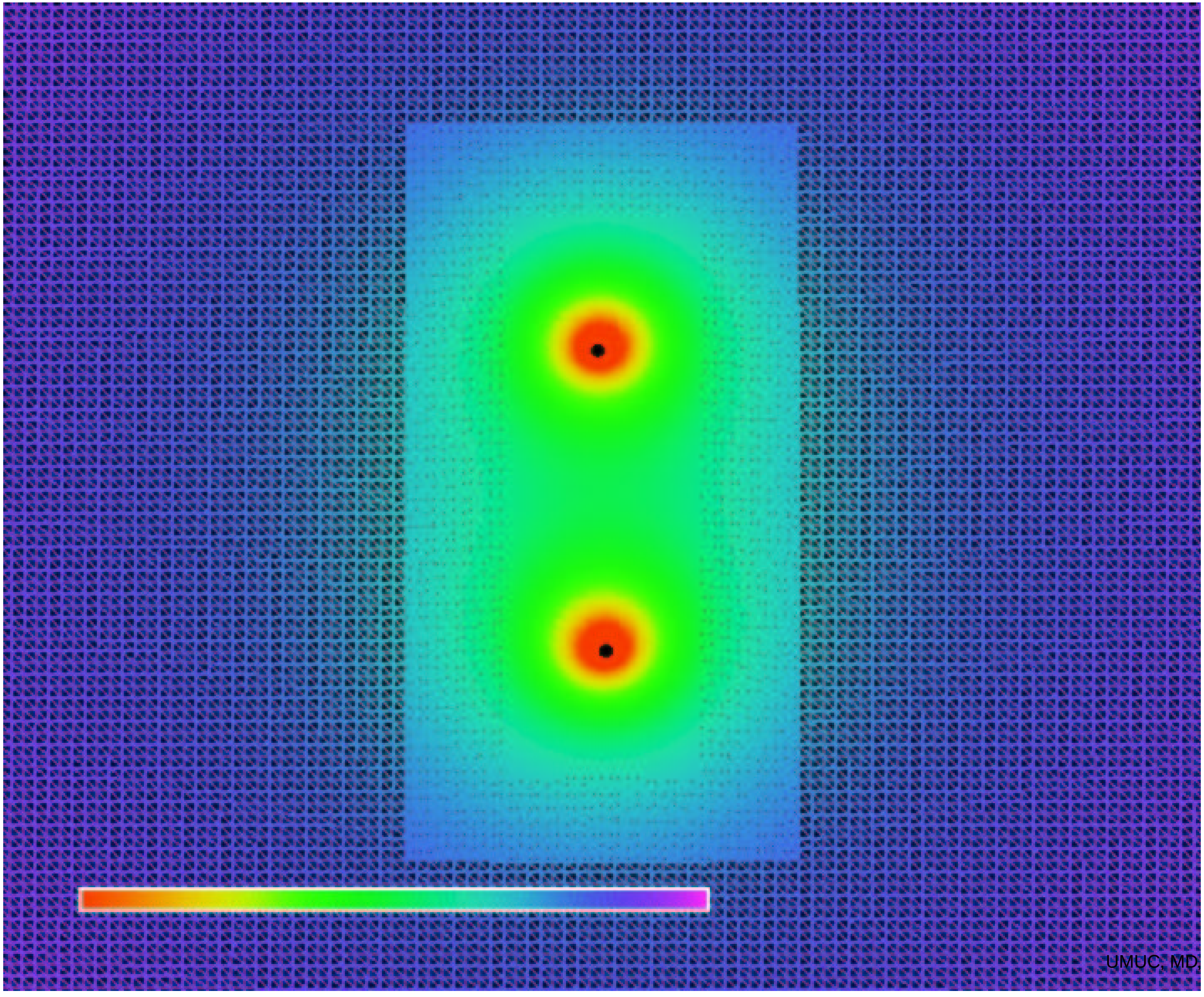

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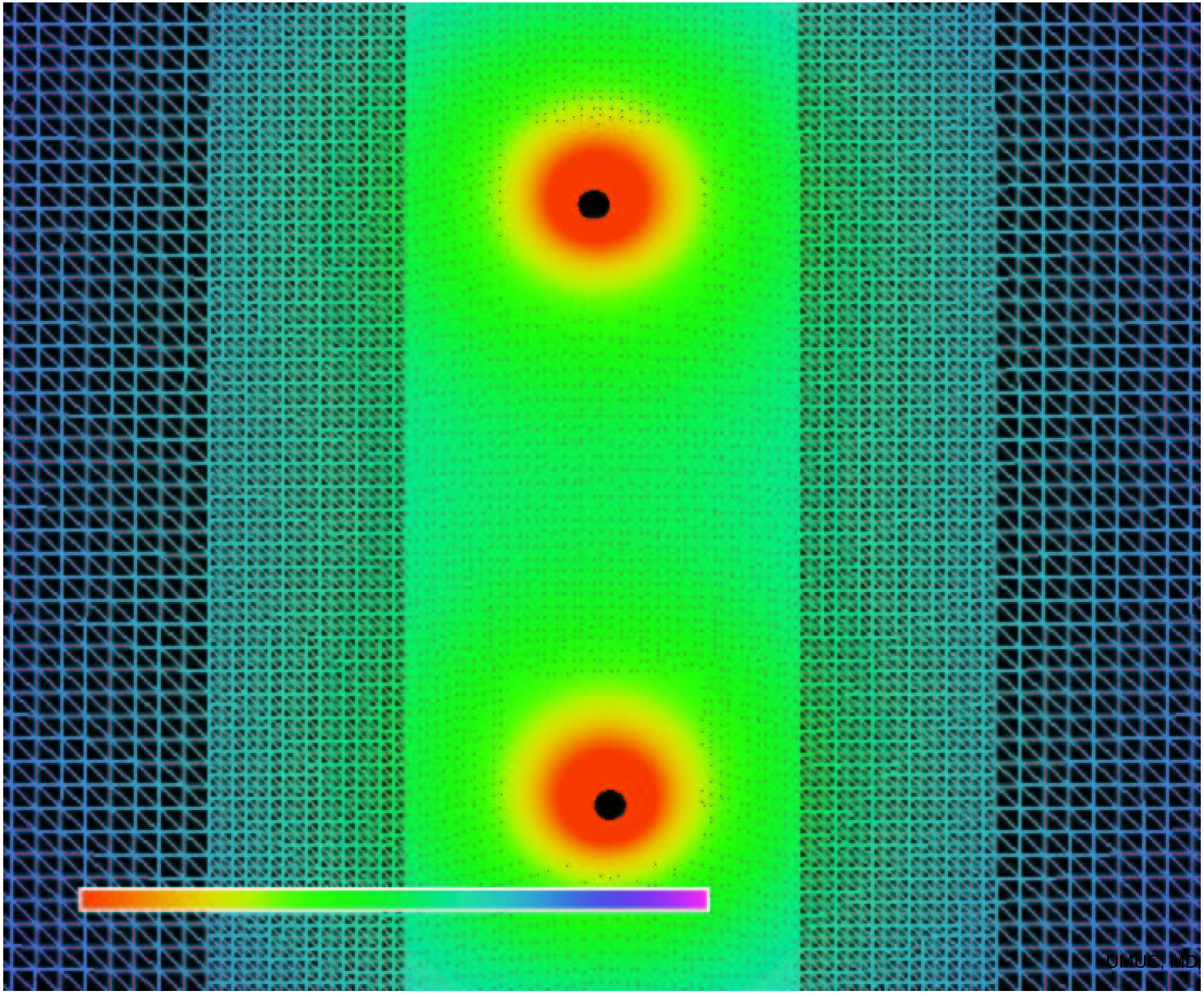
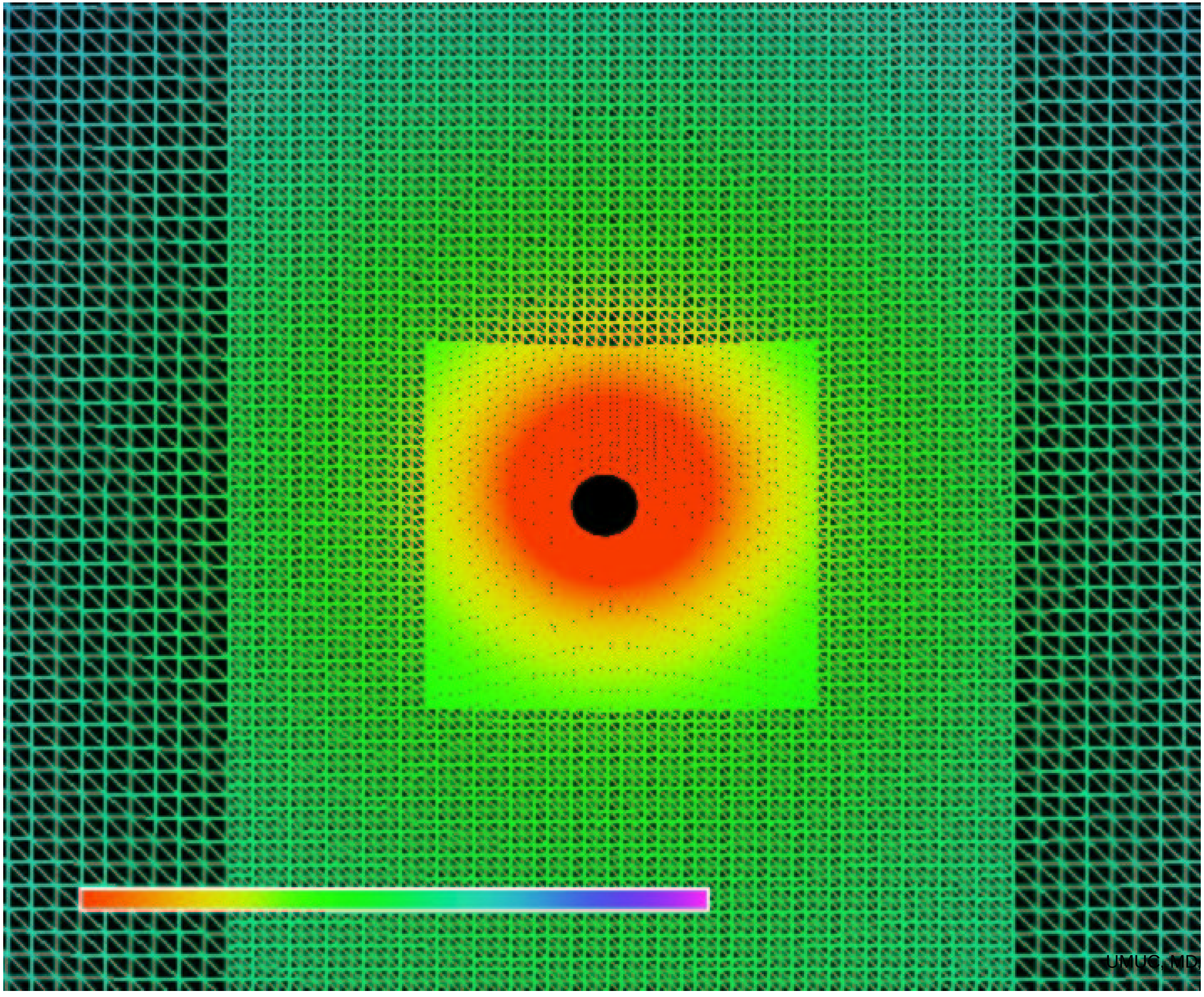
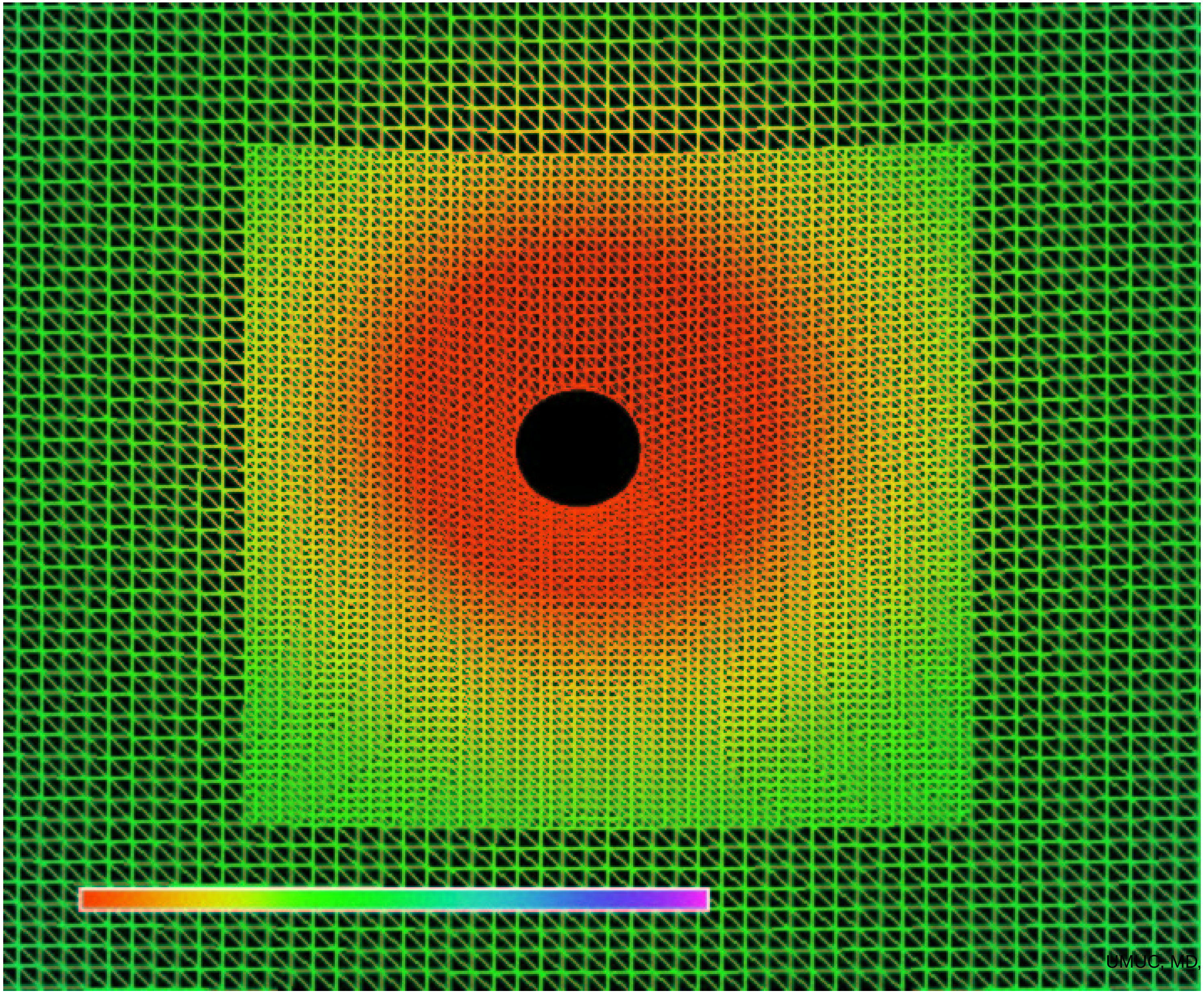


Computational grid structure



Computational grid structure



Results

- Baker, Centrella, Choi, Koppitz, van Meter, Phys. Rev. Lett. **96**, 111102 (2006)
- Baker, Centrella, Choi, Koppitz, van Meter, Phys. Rev. **D73**, 104002 (2006)
- Expect that astrophysically relevant binaries are in quasi-circular orbital configuration by the time they enter the phase where dynamics is dominated by gravitational wave emission.
- Main goals for these papers is to simulate the last few orbits of such situation. Here, we studied equal mass and non-spinning black hole binaries.

Results

- Typical simulations take $\sim 50k - 100k$ Total CPU hrs. Almost always maximize memory usage.
- $[MOVIE]$ of lapse function on $z = 0$ plane: Note black excised regions indicate roughly where the apparent
- Period of the first orbit $\sim 117M$; merges at $\sim 168M$. horizons are located.

Results: Gravitational Waveforms

- Black hole binaries (initially equal mass and non-spinning black holes) lose about $3-4\%$ of the total energy, and $25-35\%$ of the total angular momentum through gravitational wave emission.
- $\Psi_4 \sim \frac{\partial^2 h}{\partial t^2}$.
- $[MOVIE]$ of Ψ_4.

Concluding Remarks

- Computing resource via columbia/palm has been a key to our success.
- Access up to 2000 CPUs needed for our state-of-the-art simulations.
- Have access to a several hundreds of CPUs on a regular basis. Note that this is NOT the case for most of our competitions!
- Simulations results promise to have significant impacts in various directions: gravitational wave data analysis, astrophysics, test of general relativity.
- Currently working on non-equal mass and/or spinning black hole binary merger simulations.